## MATHEMATICAL STATISTICS \*

Module 2, 2024-2025

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#### 1 Course information

Course Website: https://my.nes.ru/ Instructor's Office Hours: TBD Class Time: TBD TA: TBD

#### 2 Course description

This course in Statistics is an obligatory course that is required for taking (and understanding) subsequent courses in Econometrics. Its first objective is to make sure that everyone has mastered a number of concepts such as an estimator, a statistical test, a *p*-value, as well as methods such as maximum likelihood, that are omnipresent in statistical and econometric analyses. Besides the core concepts, the course introduces a few advanced theoretical topics that may be useful both for practical work with data and for reading papers and understanding what others do. Finally, the course provide the introduction to important topics such as bayesian statistics and robust inference.

Throughout the course, we will emphasise key importance of statistics and econometrics for any solid research in economics, finance and other sciences, including that has led to Nobel Prize Awards by empirical illustrations, examples and references (see, e.g., the sample task for course evaluation below that relates to Economics Nobel Prize 2023).

<sup>\*</sup>I am grateful to Professor Andrei Savochkin for his kind permission to use his materials for the course. I also thank Professors Stanislav Anatolyev, Sergei Izmalkov and Andrei Savochkin for very helpful comments and suggestions on the course structure and materials. I also thank Professor James Stock for his kind permission to use the course materials for the Econometrics course based on SW co-taught with him at Harvard.

# 3 Course requirements, grading, and attendance policies

Student's achievements will be evaluated on the basis of problem sets, pop quizzes, and the final exam. The exam and the problem sets are graded on the 0-100 scale; the final score is computed on the same scale and then converted to transcript grades (from 2 to 5+). There will be four problem sets (possibly, of unequal size and weight) with the total weight of 24% in the final score. The remaining weight goes to the final exam. The exam score of at least 20 points is required for getting a passing grade.

The format of the exam is "A4." Each student is allowed to bring to the exam one sheet of paper of A4 size (double-sided) with notes, handwritten or typed.

#### 4 Course contents

The course covers the following topics.

- 1. General statistical methods
- Parameter estimation; Estimators for parameters of well-known distributions; Confidence intervals
- Properties of point estimators; Methods of estimation; Method of Moments.
- Maximum Likelihood method; Information inequality
- Statistical tests; Type-I and Type-II errors; Significance level and power of a test
- Likelihood ratio test
- 2. Important theoretical concepts
- Sufficient statistics; Rao-Blackwell theorem.
- Introduction to Bayesian statistics
- 3. Selected topics
- Goodness-of-fit tests
- The Analysis of Variance (ANOVA)
- Introduction to robust inference

#### 5 Description of course methodology

All course material will be presented in lectures and sections meetings. Taking notes in class is strongly recommended. Reading textbooks in addition to class attendance may be helpful but is not absolutely necessary.

#### 6 Sample task for course evaluation

(Motivated, in part, by this year's Economics Nobel Prize Award to Professor Claudia Goldin, Harvard, "for having advanced our understanding of women's labour market outcomes" and for uncovering key drivers of gender differences in the labour market. The Nobel Prize announcement, together with the Popular Background and Scientific Background to the Prize this year are available at https://www.nobelprize.org/prizes/economic-sciences/2023/press-release/).

To investigate possible gender discrimination in a US firm, a sample of 120 men and 150 women with similar job descriptions are selected at random. A summary of the resulting monthly salaries follows:

	Average Salary $(\overline{Y})$	Standard Deviation $s_Y$	n
Men	\$8200	\$450	120
Women	\$7900	\$520	150

a. What do these data suggest about wage differences in the firm? Do they represent statistically significant evidence that average wages of men and women are different? (To answer this question, first, state the null and alternative hypotheses; second, compute the relevant t-statistic; third, compute the p-value associated with the t-statistic; and, finally, use the p-value to answer the question.)

b. Do these data suggest that the firm is guilty of gender discrimination in its compensation policies? Explain.

#### 7 Course materials

#### 8 Required textbooks and materials

The majority of the topics of the course is covered in

• Hogg, R. V., J. W. McKean, and A. T. Craig (HMC), Introduction to Mathematical Statistics.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>The following is the homepage for the 8th Edition of HMC https://cs.wmich.edu/ ~mckean/hmchomepage with supplemental material (including R primer and package for the book as well as datasets). It also contains a Mathematical Primer that reviews calculus concepts used in HMC. Good classnotes on several topics covered in the monograph are available online at https://faculty.etsu.edu/gardnerr/4047/notes-Hogg-McKean-Craig.htm and https://faculty.etsu.edu/gardnerr/4047/notes-Hogg-McKean-Craig2.htm

#### 9 Additional materials

The following book can be used as an additional reading (in particular, on the topics of sufficient statistics and robust inference)

• Casella, G., and R. L. Berger, Statistical Inference.

Besides that, a lighter treatment of the core concepts with many good examples can be found in

• Hogg, R. V., E. A. Tanis, and D. L. Zimmerman, *Probability and Statistical Inference*.

The following is an excellent textbook in econometrics suggested as additional reference on the review of probability and statistics in Chs. 2 and 3. It also includes econometric applications of various concepts discussed in the course (e.g., hypothesis tests in different contexts, including tests on means and different types of regressions to be covered in econometrics courses) with many empirical examples.

• Stock, J. H. and Watson, M. W. (SW) Introduction to Econometrics.

Its online companion Introduction to Econometrics with R by C. Hanck, M. Arnold, A. Gerber, and M. Schmelzer available at https://www.econometrics-with-r. org provides (interactive) illustrations and examples for the concepts covered in SW, including those in Chs. 2 and 3 on the review of probability and statistics usin R software.

### 10 Academic integrity policy

Cheating, plagiarism, and any other violations of academic ethics at NES are not tolerated.